Equations you might need for the T206 exam

Basic Energy Equations

Energy comes in a number of different forms, and for several of these it is important to be familiar with the basic equations:

Kinetic Energy (energy of motion)

This is given by:

$$E = \tfrac{1}{2} m v^2,$$

where m is the mass of the moving object (in kg), v is its velocity (in m s^{-1}), and E is in joules (J)

In (for example) calculating the energy available to a wind turbine, we use this formula – see below for details.

Potential Energy (energy of position)

As an object moves upwards, fighting against gravitational force, some of its kinetic energy is converted into what is known as potential energy. This may be calculated thus:

$$E = m g h,$$

where m is again the mass of the object (in kg), h is the height above some convenient datum level (in m), and g is the gravitational constant at or close to the surface of the earth (approximately 9.81 m s^{-2})

In (for example) calculating the energy of falling water in a hydroelectric power station, we use this formula – see below for details. (In that instance, the head is the difference in height between the top and the bottom of the penstock.)

Electrical Energy

The energy of an electrical charge Q (in coulombs) subjected to a potential difference V (in volts) is:

$$E = Q V$$

Now current is the rate of movement of charge – i.e. charge passing a given point per unit time, so electrical power (rate of doing electrical work) is given by:

$$\text{Power} = I V$$

where the current I is measured in amperes.

This is not the only principle of electricity you need to do this course, but I will deal with the rest in a separate (probably online) document on photovoltaic cells.

Carnot Efficiency of a Heat Engine

$$\text{Efficiency} = \frac{T_H - T_C}{T_H} \times 100 = \left(1 - \frac{T_C}{T_H}\right) \times 100$$

All Temperatures must be in Kelvin.

Convert Celsius to Kelvin using 0°C = 273 K

Other Equations

It is very unlikely that you will need to use all of the following; I am reliably informed that you won't need to remember any others, though - any more complex equations that are needed will be provided in the text of the questions.

Heat loss through a wall (or a window, or...)

If the medium has an area of $\boldsymbol{A}$ (in m^2), and its U-value is (surprise surprise) $\boldsymbol{U}$ (in $Wm^{-2}K^{-1}$) and the temperature difference between inside and outside is $\boldsymbol{\Delta T}$ (in K, or, if you prefer, °C), then the rate of heat loss is:

$\boldsymbol{U A \Delta T}$ (in W)

If you can remember the units of the U-value (which is a sort of composite conductance), then you are most of the way to remembering the formula.

Heat transfer from a geothermal aquifer

Another version of the first equation, really, but here we need to express things in terms of the conductivity of the rock, $\boldsymbol{k}$ (in $Wm^{-1}K^{-1}$). The thing that you might need to calculate is the heat flux, which is the rate of upward heat transmission per unit cross-sectional area, and is normally represented by $\boldsymbol{q}$ (in Wm^{-2}). If the vertical distance through which the heat is travelling is $\boldsymbol{z}$, then the relevant equation is:

$\boldsymbol{q = k \Delta T / z}$ (in Wm^{-2})

The expression $\boldsymbol{\Delta T / z}$ is the *temperature gradient*.

Power obtained from a hydro station

Power ($\boldsymbol{P}$, in W) is expressed in terms of effective head ($\boldsymbol{H}$, in m), flowrate ($\boldsymbol{Q}$, in m^3s^{-1}), the density of water (ρ, equal to 1000 kg m^{-3}), the gravitational acceleration (g, equal to about 9.81 m s^{-2}, which for our purposes we can take as 10 m s^{-2}), and the efficiency of the turbine (η, a dimensionless value, expressed as a fraction).

In a hydro station, the potential energy of the water at the top of the penstock is converted into kinetic energy as it falls, then to mechanical and then electrical energy at the turbine. Hence we need to know how much potential energy is converted per second. Potential energy is given by:

mass x g x $\boldsymbol{H}$

so potential energy converted per second is given by:

mass flowrate x g x $\boldsymbol{H}$

Now mass flowrate (in kg s^{-1}) is the same as (density of water x volume flowrate) or $\rho \boldsymbol{Q}$

so power input to turbine = $\rho \boldsymbol{Q} g \boldsymbol{H}$

with the result that power output from turbine is:

$\boldsymbol{P} = \eta \rho \boldsymbol{Q} g \boldsymbol{H}$

or, taking ρ =1000 and (approximating somewhat) g = 10,

$\boldsymbol{P} = 10000\, \eta \boldsymbol{Q H}$ (if power is measured in W) or (power out <u>in kW</u>) = $10\ \eta\ \boldsymbol{Q}\ \boldsymbol{H}$

Power obtained from a wind turbine

Power ($\boldsymbol{P}$, in W) is expressed in terms of wind velocity ($\boldsymbol{V}$, in ms^{-1}), the density of air (ρ, equal to about 1.2 kg m^{-3}), the swept area of the turbine ($\boldsymbol{A}$, in m^2), and the efficiency of the turbine (η, a dimensionless value, expressed as a fraction).

In a wind turbine, the kinetic energy of the air approaching the turbine is converted into mechanical and then electrical energy. Hence we need to know how much kinetic energy is converted per second. Kinetic energy is given by:

$\frac{1}{2}$ x mass x $\boldsymbol{V}^2$

so kinetic energy converted per second is given by:

$\frac{1}{2}$ x mass flowrate x $\boldsymbol{V}^2$

Now mass flowrate (in kg s^{-1}) is the same as (density of air x volume flowrate), which is (density of air x velocity of air x cross-sectional area of the bit of air we're concerned with), which is $\rho\ \boldsymbol{V}\boldsymbol{A}$

so power input to turbine $= \frac{1}{2} \text{ x } \rho\ \boldsymbol{V}\boldsymbol{A} \text{ x } \boldsymbol{V}^2 = \frac{1}{2}\rho\ \boldsymbol{A}\ \boldsymbol{V}^3$

with the result that the power *output* is:

$\boldsymbol{P} = \frac{1}{2}\ \eta\ \rho\ \boldsymbol{A}\ \boldsymbol{V}^3$

Chemical equations for combustion of different fuels

It is conceivable (but, I think, unlikely) that you'll have to figure out the mass of carbon dioxide emitted from fossil and/or bio- fuels. You can do this from the chemical equations concerned, e.g. for methane,

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

so one mole of methane yields one mole of carbon dioxide. This means that (given relative atomic masses of C=12, H=4, O=16) 16kg methane yields 44kg carbon dioxide - i.e. (44/16) kg CO_2 per kg methane burnt. If you are then told that 1kg methane has an energy content of 55MJ, you can calculate that it produces:

(44/(16x55)) = 0.05 kg CO_2 per MJ out (or 50 kg/GJ).

If you end up having to do this, there will be some assistance in the question, so don't be overly anxious if you can't remember much chemistry. Just try to understand the steps involved in the argument above, and you'll be OK.